**THE EUROPEAN
PHYSICAL JOURNAL C**

QCD corrections to QED vacuum polarization

P. Castelo Ferreira[a], J. Dias de Deus[b]

CENTRA, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Abstract. We compute QCD corrections to QED calculations for vacuum polarization in background magnetic fields. Formally, the diagram for virtual $e\bar{e}$ loops is identical to the one for virtual $q\bar{q}$ loops. However, due to confinement, or to the growth of α_s as p^2 decreases, a direct calculation of the diagram is not allowed. At large p^2 we consider the virtual $q\bar{q}$ diagram, in the intermediate region we discuss the role of the contribution of quark condensates $\langle q\bar{q} \rangle$ and at the low-energy limit we consider the π^0, as well as charged pion $\pi^+\pi^-$ loops. Although these effects seem to be out of the measurement accuracy of photon–photon laboratory experiments, they may be relevant for γ-ray burst propagation. In particular, for emissions from the center of the galaxy (8.5 kpc), we show that mixing between the neutral pseudo-scalar pion π_0 and photons renders a deviation from the power-law spectrum in the TeV range. For scalar quark condensates $\langle q\bar{q} \rangle$ and virtual $q\bar{q}$ loops are relevant only for very high radiation density $\sim 300\,\mathrm{MeV/fm}^3$ and very strong magnetic fields of order $\sim 10^{14}$ T.

PACS. 12.20.Ds; 14.40.-n; 12.38.Aw; 14.65.Bt

1 Introduction

In the presence of background electromagnetic fields second order QED corrections in the fine-structure constant α to vacuum polarization due to quantum vacuum oscillations, i.e. electron–positron virtual loops [1, 2], is a well studied subject, including Delbrück scattering [3], photon splitting [4, 5], photon–photon interactions [6, 7] and semi-classical interactions with pseudo-scalar particles [8–11]. QED effects are well established, and Delbrück scattering has been experimentally observed in light scattering by heavy nuclei [12] as well as contributing in second order in αZ to the Lamb shift [13, 14]. However, a direct signature for the interaction with axion-like pseudo-scalars has not been consistently verified [15–20]. Independent experiments to detect this sort of interaction have now been considered [21–24]. In addition, in astro-particle observations, due to the wide range of energies accessible, QED effects are the main contribution for the optical depth. Such effects include particle production (also known as photon disintegration) [25–36], vacuum polarization and photon splitting [37–40]. Astro-particle observation also seems to be the most promising procedure to test pseudo-scalar interactions [41–52] through, for example, analysis of gamma-ray burst conversion rates [53, 54] and its polarization characteristics [55–57].

In this work we study the second order perturbative corrections in the fine-structure constant due to the strong interactions. Namely, we analyze quark loops, quark condensates and meson contributions to vacuum polarization.

The second order contribution to the polarization of the vacuum due to electron–positron virtual pair production is given by the Euler–Heisenberg Lagrangian [1, 2]

$$
\begin{aligned}
\mathcal{L}_{e\bar{e}}^{(2)} &= \xi_e \left[4\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + 7\left(\epsilon^{\mu\nu\delta\rho}F_{\mu\nu}F_{\delta\rho}\right)^2 \right], \\
\xi_e &= \frac{2\alpha}{45(B_c^e)^2} = 1.32 \times 10^{-24}\,\mathrm{T}^{-2}, \\
B_c^e &= \frac{m_e^2 c^2}{e\hbar}.
\end{aligned}
\tag{1}
$$

The respective vacuum polarization dispersion relation for radiation propagating in vacuum under an orthogonal magnetic field $B_0 \ll B_c^e$ is expressed in terms of the eigenvalues in the cases of being orthogonal and parallel to the magnetic field [4, 5],

$$
\begin{aligned}
\omega_{\perp,\parallel} &= \left(1 - \lambda_{\perp,\parallel}^e B_0^2\right), \\
\lambda_\perp^e &= 8\xi_e B_0^2, \quad \lambda_\parallel^e = 14\xi_e B_0^2.
\end{aligned}
\tag{2}
$$

Also within the framework of QED we have the contribution of higher mass fermionic loops. Relevant to the present study we have the muon loops $\mu\bar{\mu}$, which give the contribution

$$
\begin{aligned}
\lambda_{\perp,\parallel}^\mu &= \Delta\xi_\mu \lambda_{\perp,\parallel}^e, \\
\Delta\xi_\mu &= \frac{\xi_\mu}{\xi_e} = \left(\frac{m_e}{m_\mu}\right)^4 = 5.43 \times 10^{-10},
\end{aligned}
\tag{3}
$$

[a] e-mail: pedro.castelo.ferreira@ist.utl.pt
[b] e-mail: jdd@fisica.ist.utl.pt

being of the same order of magnitude of the QCD corrections that we are addressing here. Due to its higher mass the τ gives a contribution five orders of magnitude lower ($\sim 6 \times 10^{-15}$), hence not being relevant here.

2 QCD contributions

Naively, we can expect that the same kind of physics applies to quark–antiquark virtual pair production. In the presence of an external field we have in general the diagram of Fig. 1. We write in the case of $q\bar{q}$ virtual pair production in order α^2 the Euler–Heisenberg Lagrangian

$$\mathcal{L}_{q\bar{q}}^{(2)} = \xi_q \left[4 \left(F_{\mu\nu} F^{\mu\nu} \right)^2 + 7 \left(\epsilon^{\mu\nu\delta\rho} F_{\mu\nu} F_{\delta\rho} \right)^2 \right] ,$$
$$\xi_q = \delta_q \frac{2\alpha Q_q^2}{45 \left(B_c^q \right)^2}, \quad B_c^q = 3 \frac{m_q^2 c^2}{e \, Q_q \, \hbar} . \quad (4)$$

The factor of 3 comes from the summation over colors and Q_q stands for the fractional charge of the quark. The quark masses, m_q, correspond to the *renormalized* masses that appear in the quark propagator. Under an external magnetic field B_0 the relation of the polarization due to $q\bar{q}$ with the polarization due to $e\bar{e}$ corresponding to electron–positron loops is given in terms of the corrections to the parallel and orthogonal vacuum dispersion relation eigenvalues $\lambda_\perp^q$ and $\lambda_\parallel^q$ by

$$\lambda_{\perp,\parallel}^q = \Delta\xi_q \lambda_{\perp,\parallel}^e ,$$
$$\Delta\xi_q = \frac{\xi_q}{\xi_e} = 3\delta_q \left(\frac{m_e Q_q}{m_q} \right)^4 \approx 6.41 \delta_q \times 10^{-4} . \quad (5)$$

Here we consider the up quark mass $m_q \approx 5\,\text{MeV}$ and charge $Q_q = 2/3$, and $\delta_q = w_{\Lambda_q}/w_{\text{tot}} < 1$ is a phase space correction due to confinement of strong interactions. We know that at low energies there are no free quarks; therefore, quark loops carrying small momenta cannot be considered. The way out is to introduce a lower cut-off Λ_q in

the loop momenta such that only the higher momenta contributions to the loop are considered.

The probability for the full range of momenta (i.e. $p^2 \in]0, +\infty[$) is given by the series [58]

$$w_{\text{tot}} = \frac{\alpha \mathbf{B}^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n\pi m_q^2}{|Q_q \mathbf{B}|}} . \quad (6)$$

Due to confinement and the increase of α_s for small values of p^2, we introduce a cut-off Λ_q that truncates the series (6) by excluding the low p^2 region. We have

$$w_{\Lambda_q} = \frac{\alpha \mathbf{B}^2}{\pi^2} \sum_{n=n_{\Lambda_q}}^{\infty} \frac{1}{n^2} e^{-\frac{n\pi m_q^2}{|Q_q \mathbf{B}|}} . \quad (7)$$

We obtain the relation

$$n_{\Lambda_q} = \left(\frac{\Lambda_q}{m_q} \right)^2 . \quad (8)$$

For the light quarks (summing over up and down quark masses) with mass of order $m_q \sim 10\,\text{MeV}$ [59] we have $n_{\Lambda_q} \sim 3600$ such that the exponent is of order $\delta_q \sim 10^{-10^{14}/|B|}$. This value is obtained from the leading term contribution from the above series for w_{Λ_q}. Here we considered $\Lambda_q \sim 600\,\text{MeV}$, this being the value for which the strong interactions coupling constant becomes unity, $\alpha_s \sim 1$ [60–64], so that below this energy threshold, QCD is in a non-perturbative regime. The free quark loop contribution to vacuum polarization is therefore negligible. This contribution will only be relevant for very strong magnetic fields of order $B \sim 10^{14}\,\text{T}$.

The low-energy quark states (corresponding to the light mesons) are the π. In low-energy physics these particles can be used as *fundamental* bosons within the framework of chiral perturbation theory (ChPT) [65–75]. Therefore, below the cut-off $p^2 < \Lambda_q$ the main contribution is due to the neutral meson π^0 with an effective scalar Lagrangian [2]:

$$\mathcal{L}_{\pi^0}^{(2)} = \frac{1}{4} g_{\pi\gamma\gamma} \phi_{\pi^0} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} ,$$
$$g_{\pi\gamma\gamma} = \frac{\alpha}{\pi f_\pi} = 2.49 \times 10^{-2}\,\text{GeV}^{-1} , \quad (9)$$

where the coupling $g_{\pi\gamma\gamma}$ is taken from the Adler–Bell–Jackiw anomaly coefficient [76] and the pion decay constant is taken to be $f_\pi = 93\,\text{MeV}$. The respective contribution to the dispersion relation of radiation traveling in vacuum (corresponding to the π diagram of Fig. 1) is

$$\lambda_\perp^{\pi^0} = 0 , \quad \lambda_\parallel^{\pi^0} = \Delta\xi_{\pi^0} \lambda_\parallel^e ,$$
$$\Delta\xi_{\pi^0} = \frac{g_{\pi^0\gamma\gamma}^2}{14\xi_e} = \frac{45 m_e^4}{14\pi^2 m_\pi^2 f_\pi^2} = 1.40 \times 10^{-10} . \quad (10)$$

We considered the pion mass $m_\pi = 135\,\text{MeV}$ again; for higher masses the contributions are for most applications negligible when compared to the π^0 effect.
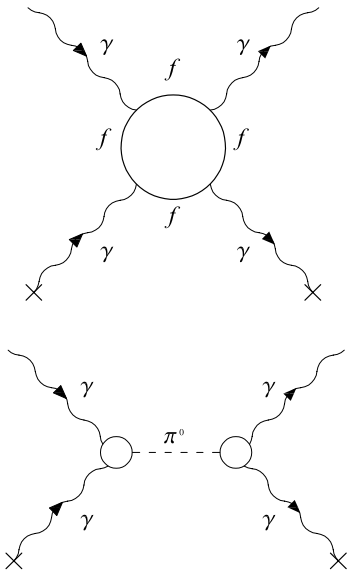
Fig. 1. The diagrams for fermion–antifermion loops and the exchange of a π^0 neutral meson. The vertex $\pi^0\gamma\gamma$ includes the axial anomaly

In addition we can have loops of the lighter charged mesons $\pi^+\pi^-$ whose Euler–Heisenberg Lagrangian reads [2]

$$\mathcal{L}_{\pi^+\pi^-}^{(2)} = \xi_{\pi^\pm}\left[7\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + 4\left(\epsilon^{\mu\nu\delta\rho}F_{\mu\nu}F_{\delta\rho}\right)^2\right],$$

$$\xi_{\pi^\pm} = \frac{\alpha}{45\left(B_c^e\right)^2}, \quad B_c^e = \frac{m_\pi^2 c^2}{e\hbar}, \tag{11}$$

hence contributing a correction to the vacuum dispersion relation of approximately the same order of magnitude given by

$$\lambda_\perp^{\pi^\pm} = \frac{7}{4}\Delta\xi_{\pi^\pm}\lambda_\perp^e, \quad \lambda_\parallel^{\pi^\pm} = \frac{4}{7}\Delta\xi_{\pi^\pm}\lambda_\parallel^e$$

$$\Delta\xi_{\pi^\pm} = \frac{\xi_\pi}{\xi_e} = \frac{1}{2}\left(\frac{m_e f_\pi}{m_\pi^2}\right)^4 = 2.29\times10^{-11}. \tag{12}$$

There is yet another contribution that we can consider. In the presence of background magnetic fields there is a vacuum polarization contribution due to quark condensates. Within the Schwinger–Euler–Heisenberg formalism [1, 2] in the context of ChPT [77–79] a correction to the vacuum dispersion is obtained:

$$\lambda_\perp^c = \Delta\xi_c\lambda_\perp^e, \quad \lambda_\parallel^c = 0,$$

$$\Delta\xi_c = \frac{\xi_c}{8\xi_e} = \frac{15m_e^4}{128f_\pi^4}\ln\left(\frac{\Lambda^2}{m_\pi^2}\right)$$

$$= 1.05\times10^{-10}\ln\left(\frac{\Lambda^2}{m_\pi^2}\right). \tag{13}$$

Taking the quark condensate ultra-violet cut-off $\Lambda\approx 300\,\mathrm{MeV}$ we obtain $\Delta\xi_c \approx 1.69\times10^{-10}$. Next we give some details on how quark condensates are obtained and explain what regimes exist depending on the loop momentum. The parallel vacuum polarization for ChPT is given by the integral

$$\Pi_{\langle q\bar{q}\rangle} = \int_0^\infty \mathrm{d}s\, I_{\langle q\bar{q}\rangle},$$

$$I_{\langle q\bar{q}\rangle} = -\frac{\alpha B}{12f_\pi^4}\frac{1}{s^2}\left[\alpha B\cot(\alpha Bs) - \frac{1}{s}\right]. \tag{14}$$

This distribution is represented in Fig. 2. The contributions considered here are due to the poles below the cut-off $s < 1/\Lambda^2$. For weak fields the only pole that contributes for pion loops is at $s = 0$. It corresponds to the $\pi^+\pi^-$ loops and the relative magnitude of its effect has already been discussed and is given in (12). Above the cut-off $s > 1/\Lambda^2$ we consider the quark loops instead of the meson distributions. The novel interesting feature in this framework is that we have a new contribution between the pole $s = 0$ and $s = \pi/\alpha|B|$; it corresponds to the quark condensate. We note that on a more fundamental level based on Nambu–Jona-Lasinio theory [80, 81] the quark condensates contribution is of the same order of magnitude [82–88]. There is an important point to stress. NJL consider explicit actions for the quarks instead of the effective actions for the mesons considered in ChPT. The condensate cut-off Λ should correspond according to NJL to the confinement
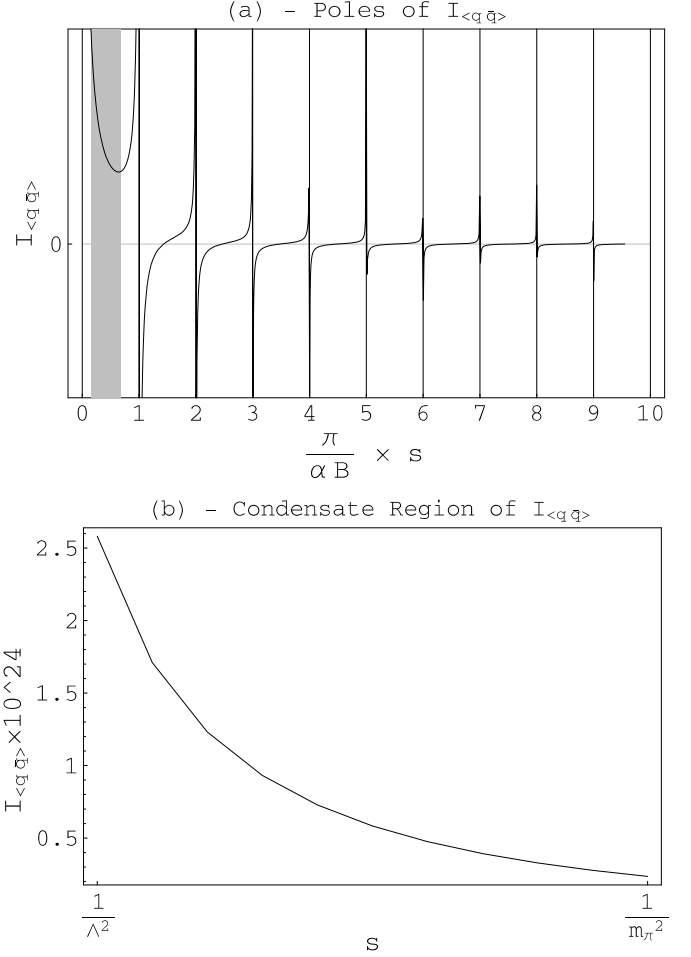
Fig. 2. **a** The integrand (14). The poles at $s = (n-1)\pi/\alpha B$ (for $n = 1, 2, \ldots, \infty$) are marked by *vertical lines* and contribute to the pion vacuum polarization. **b** The same integrand between the two cut-offs $m_\pi = 135\,\mathrm{MeV}$ and $\Lambda = 300\,\mathrm{MeV}$ for $B = 5.5\,\mathrm{T}$. It corresponds to the marked region between the poles at $s = 0$ and $s = \pi/\alpha B$ of **a**

energy. These theories were originally motivated by superconductivity, and the relation between ChPT and NJL is equivalent to the relation between Landau–Ginzburg effective theory [89] and the Bardeen–Cooper–Schrieffer microscopic theory [90–92] for superconductivity.

We summarize the allowed effects and their magnitude for several ranges of loop momenta p in Table 1.

We note that the π_0 is a 0^{-+} pseudo-scalar such that CP-symmetry is conserved; the Lagrangian (9) is a scalar. More generally we may consider also the contributions of other 0^{-+} pseudo-scalars like the η; however, their masses are higher than the mass of the pion (the lighter one being $m_\eta \approx 547\,\mathrm{MeV}$); hence, their contribution is negligible by several orders of magnitude.

Quark condensates are 0^{++} scalars. Although we have already presented the effects of these condensates in (13) obtained within the mean-field framework of ChPT, we note that we can recast these effects diagrammatically considering the scattering of photons by an intermediate scalar. However, if one demands CP-invariance the ef-

Table 1. The several QCD effects in the presence of weak fields and their magnitude for the several ranges of the loop momenta p. To exist, the quark condensate requires a high density of energy

p (MeV)	$\Delta\xi_{\pi^0}$	$\Delta\xi_c$	$\Delta\xi_q$
> 600	0	0	$10^{(-10^{14})}$
140–600	0	9.67×10^{-11}	0
< 140	1.40×10^{-10}	0	0

fective action is expressed as $\mathcal{L} = g_{\langle q\bar{q}\rangle}\phi_{\langle q\bar{q}\rangle}F^{\mu\nu}F_{\mu\nu}$, where $\phi_{\langle q\bar{q}\rangle}$ now represents a scalar. In the same manner one may consider the other 0^{++} scalar mesons f_0 and a_0 [93]. Although these processes are allowed and the couplings are of the same magnitude as of the pion, they are in both cases negligible due to the much higher masses [59].

In addition, we also note that the quark condensates may only exist when very high densities of energy are present $\langle E\rangle \sim 300\,\text{MeV}/\text{fm}^3$ [77–79]. These values are only accessible in very dense plasmas (for example in neutron stars [94–105]) or for very high fluxes of high energy radiation. As an example for radiation energies in the TeV range, propagating in vacuum, fluxes are required over 10^{56} photons $\text{m}^{-2}\text{s}^{-1}$. It is understood that these fluxes include both the propagating and the background photons.

3 Vacuum birefringence

The relevant radiative corrections that contribute to vacuum birefringence are of second order in the fine-structure constant [4, 5]. The usual classical wave equation in order α^2 is linear in the photon field A [5]. Hence for a static transverse magnetic field B_0, gathering the results from the previous section, we obtain, due to QCD effects, a correction on the refractive index eigenvalues given by [4, 5, 21–23]:

$$\lambda_\perp = 8\left(1 + \Delta\xi_\mu + \Delta\xi_c + \frac{7}{4}\Delta\xi_{\pi^\pm}\right)\xi_e B_0^2,$$

$$\lambda_\parallel = 14\left(1 + \Delta\xi_\mu + \Delta\xi_{\pi^0} + \frac{4}{7}\Delta\xi_{\pi^\pm}\right)\xi_e B_0^2. \quad (15)$$

The directions $\parallel$ and $\perp$ correspond respectively to the directions parallel and transverse to the external magnetic field, ξ_e is given in (2), and the several corrections are ordered by magnitude significance according to the estimate given in (3), (10), (12) and (13). The above equations result in different refractive indexes for the directions parallel and perpendicular to the magnetic field [21–23]:

$$N_\parallel = 1 - \frac{1}{2}\lambda_\parallel, \quad N_\perp = 1 - \frac{1}{2}\lambda_\perp, \quad (16)$$

which introduce a phase shift in the propagating wave. Considering a linearly polarized wave of wave number $\mathbf{k} = k_0\mathbf{z}$, which polarization makes an angle θ_0 with the static magnetic field B_0, both gain an ellipticity, and

its polarization is rotated due to vacuum effects (see for example [21–23]). The polarization rotation is given by

$$\Delta\theta = \frac{1}{4}\left(\lambda_\parallel - \lambda_\perp\right)\Delta z \sin(2\theta_0), \quad (17)$$

Δz being the distance traveled by the radiation, and the ellipticity is given by

$$\psi = -\frac{\omega}{4}\left(\lambda_\parallel - \lambda_\perp\right)\Delta z \sin(2\theta_0), \quad (18)$$

ω being the radiation frequency. The relative magnitudes of rotation induced in vacuum by the several effects presented here are pictured in Fig. 3. We note that the contribution to the rotation of the pseudo-scalars have the same sign of the fermionic loops contributions ($\lambda_\parallel > \lambda_\perp$), while the contribution due to the charged pion loops have an opposite sign ($\lambda_\parallel^{\pi^\pm} < \lambda_\perp^{\pi^\pm}$).

We note that the usual QCD scale is set by $\Lambda_{\text{QCD}} \approx 200\,\text{MeV}$; however, as already discussed, we also know that for energies of approximately $\Lambda_q = 600\,\text{MeV}$ the strong running coupling constant α_s is of order of unity and the perturbative regime of QCD is no longer valid [60–64]. Therefore the correct value of the cut-off corresponding to low-energy quark condensate is not exactly known and should be in the range $200 < \Lambda < 600\,\text{MeV}$. This value should correspond to the chiral phase transition energy of the Nambu–Jona–Lasinio theory [80, 81].

As already put forward in [9] (see also [10, 11]) these effects, in particular the contribution of the pion, is several orders of magnitude lower than the one due to the electron
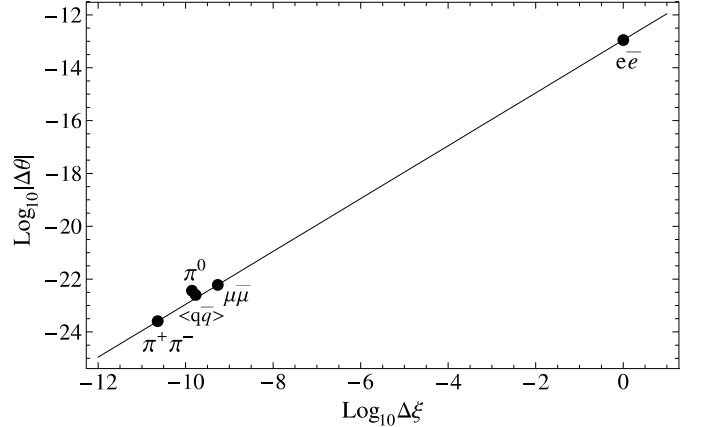
Fig. 3. Relative contributions to the polarization rotation as given by (17) from the several effects as a function of $\Delta\xi_i$, i.e. each effect's magnitude in relation to the magnitude of the effect due to electron–positron loops ($e\bar{e}$). Both axes are in logarithmic scale. The *continuous line* coincides approximately with the PVLA experimental conditions [15–19] with $B_0 = 5.5\,\text{T}$ and $\Delta z = 10^9\,\text{m}$. The *marked points* are labeled and correspond to the QED corrections due to electron–positron loops ($e\bar{e}$), the muon–antimuon loops ($\mu\bar{\mu}$) interchange of the neutral pion (π^0), quark condensates ($\langle q\bar{q}\rangle$) and the charged pion loop ($\pi^+\pi^-$). The ellipticities are obtained by re-scaling these results by the radiation frequency ω

vacuum oscillation as well as of the theoretical axion contribution. In addition, we recall that the quark condensate contribution is only present if a very high density of radiation is considered.

We conclude that vacuum birefringence due to QCD corrections is negligible for all known physical systems. As already stated in [9], the main contribution is due to the neutral pion being many orders of magnitude below the vacuum polarization effects of virtual electron loops, which, although a well established phenomenon within QED, has not directly been detected by any polarization rotation, neither in laboratory experiments, nor in astrophysical environments. However, in the next section we give an example where the light scattering by the π^0 meson may have measurable effects.

4 High energy γ-ray propagation

We can also apply the results derived so far to the propagation of γ-ray bursts in background magnetic fields where the effects studied in the previous sections seem to be relevant due to the very high radiation energy. The main contribution to the attenuation of the γ-ray spectrum for high energies (E above 10^{16} eV) is photon disintegration (particle–antiparticle pair production) due to interaction with the background electromagnetic fields [25–36]. These effects result in an exponential decaying law for the γ-ray spectrum, $\sim E^{-\Gamma(z)}$. For radiation from the center of the galaxy (corresponding to $z = 8.5$ kpc) the value of the decaying exponent is $\Gamma \approx 2.25$ [53, 54]. The radiation flux is of order 10^{-8} photons m^{-2} s^{-1} for photons above the TeV range [41–46] (corresponding to a radiation density of order $\rho_\gamma \sim 10^{-54}$ MeV/fm^3). Therefore, for this particular case, the only relevant contribution discussed in this work is from the π^0 meson, which we address next.

We note that it is also expected that the axion-like pseudo-scalar contribution has visible effects in the high energy range (of order of TeV), either increasing or decreasing the optical depth depending on the specific values of the mass and photon coupling constant considered. More specifically it is expected that for more stable particles with long decaying time (low decaying rate) like the light axion, the optical depth increases [9, 47–52], while for less stable particles with lower decaying time (greater decaying rate), the optical depth decreases. What distinguishes these cases is the relation of the pseudo-scalar mass (m_ϕ) to the photon pseudo-scalar coupling ($g_{\gamma\phi}$) properly taking in consideration the background and traveling radiation energy. We will return to this discussion by the end of this section. Specifically the equation for a generic pseudoscalar ϕ mixing with the photon is [9, 47–52]

$$(\omega - \mathrm{i}\partial_z + M) \begin{bmatrix} A_\parallel \\ A_\perp \\ \phi \end{bmatrix} = 0 , \qquad (19)$$

with

$$M = \begin{bmatrix} \Delta_{\gamma\gamma} + \Delta_\parallel & 0 & \Delta_{\gamma\phi}^\parallel \\ 0 & \Delta_{\gamma\gamma} + \Delta_\perp & \Delta_{\gamma\phi}^\perp \\ \Delta_{\gamma\phi}^\parallel & \Delta_{\gamma\phi}^\perp & \Delta_\phi \end{bmatrix} , \qquad (20)$$

and the various entries given by

$$\Delta_{\gamma\gamma} \approx -\mathrm{i}\frac{\Gamma}{2z_0} \ln(E) ,$$
$$\Delta_\parallel \approx 4\xi_e B^2 , \quad \Delta_\perp \approx 7\xi_e B^2 ,$$
$$\Delta_{\gamma\phi}^{\parallel,\perp} = \frac{1}{2}g_{\gamma\phi}B^{\parallel,\perp} , \quad \Delta_\phi = m_{\pi_0} . \qquad (21)$$

The approximation in $\Delta_{\gamma\gamma}$ corresponds to a linearization of the cross section for photon disintegration in the TeV range of the energies (E) for distances $z_o \approx 8.5$ kpc and we have considered the photon mass negligible ($m_\gamma \approx 0$). The approximation in $\delta_{\parallel,\perp}$ corresponds to neglecting the contributions from $\mu\bar{\mu}$ loops and the charged pion $\pi^+\pi^-$ loops. As derived in [50–52], for non-polarized radiation in Gaussian magnetic field distributions, the conversion probability of photons to pseudo-scalars is

$$P_{\gamma\to\phi} = \frac{1}{3} \left(1 - \mathrm{e}^{-\frac{3P_0 z}{2s}} \right) , \qquad (22)$$

$$P_0 \approx 0.4 \times 10^{-7} \left(\frac{gB_{\mathrm{G}}E_{10}}{m_\phi^2} \right)^2 . \qquad (23)$$

These expressions are applicable to the mixing of the neutral pseudo-scalars to photons in γ-ray bursts with coupling given by $g = g_{\gamma\phi}/10^{-6}$ GeV^{-1}, the root mean square magnetic field strength $B_{\mathrm{G}} = 1 \mu$Gauss, the energy E_{10} given in units of 10 TeV and the mass given in MeV. z is the distance to the source in pc and s the size of the magnetic field domains also in pc [50–52]. For a γ-ray burst from the center of the galaxy one has $z = 8.5$ kpc and $s = 0.01$ pc [50–54]. For the particular cases of π_0 mixing we have $g = 2.49 \times 10^4$ and $m = 135$ MeV. The deviation to the power law considering this effect is pictured in Fig. 4.

As we have put forward in the beginning of this section, depending on the specific coupling constants, masses and existing energies, the effects of the pseudo-scalar mixing to photons renders quite different results. The relevant expressions to compare correspond to the diagonal and off-diagonal components of the matrix for pseudo-scalar photon mixing. Respectively the relevant expressions to compare are $m_\phi^2 z/s$ and $g_\phi BE$. This analysis was originally in [9] and was also considered in [47–52]. The main differences between these several works are the numerical values for the parameters of the propagation equation. While in [9] the length z/s traveled by radiation is of order of km, in [50–52] astrophysical environments are considered with traveling over several kpc, which allows the scalar–photon oscillations to saturate as given by (23). The quantitative similarities of our results in relation to [50–52] is simply due to the ratios of the couplings to the mass squared of the *heavy* axion considered there and the pion considered here,
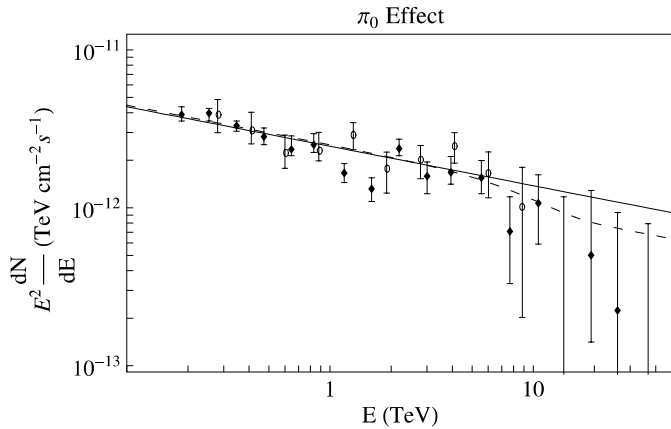
Fig. 4. Deviations from the power law due to π_0 mixing with photons. The *open circles* and *filled circles* are data points from the HESS collaboration corresponding to the data sets from July/August of 2003 and 2004 [53, 54] (see also [50–52]). The *filled line* represents the power law best fit $dN/dE \sim E^{-\Gamma}$ with $\Gamma = 2.25$ and the *dashed lines* corresponds to the contributions of the π_0

this being different only by about one order of magnitude $21 \times g_\pi/m_\pi^2 \approx g_{\text{axion}}/m_{\text{axion}}^2$. As for [47–49], we note that in this work a very light axion $m_{\text{axion}}^2 \ll g_{\gamma\,\text{axion}}EB$ is considered and calculations are carried out in the massless limit, $m_{\text{axion}} \to 0$. This latter case is clearly not applicable to the pion, where we have $m_\pi^2 > g_{\gamma\pi}BE$, and this explains why for a relatively heavy intermediate pseudo-scalar (with low decaying times) the optical depth is decreased while for a very light intermediate pseudo-scalar (with high decaying times) the optical depth is increased.

One could also consider the more generic case of several pseudo-scalar and scalar mixing; not only the theoretically suggested axion, but also the quark condensate effects when the conditions for their formation are met (for example close to neutron stars and magnetars [94–105]).

5 Conclusions

In this work we have computed the QCD corrections to QED vacuum polarization effects. Although we conclude that the contribution to vacuum birefringence of the effects presented here are negligible when compared to the effect of virtual electron loops, they have observable consequences for high energy γ-ray propagation. In particular, we have shown that the neutral pion mixing with photons significantly contributes to a deviation from the power-law spectrum in the TeV range, which may be relevant when considering the superposition of other pseudo-scalar effects in this range [41–46, 50–52]. As for quark condensates and virtual quark loops, we conclude that only for very high radiation energy fluxes ($\rho_c > 300\,\text{MeV}/\text{fm}^3$) and strong magnetic fields ($B > 10^{14}\,\text{T}$), their effects may be relevant. Hence near neutron stars and magnetars [94–105], these effects may affect γ-ray polarization [55–57].

References

1. W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936) [physics/0605038]
2. J. Schwinger, Phys. Rev. **82**, 664 (1951)
3. M. Delbrück, Z. Phys. **84**, 144 (1933)
4. Z. Bialynicka-Birula, I. Bialynicka-Birula, Phys. Rev. D **2**, 2341 (1970)
5. S.L. Adler, Ann. Phys. **67**, 559 (1971)
6. E. Lundström et al., Phys. Rev. Lett. **96**, 083602 (2006)
7. M. Marklund, P.K. Shukla, Rev. Mod. Phys. **78**, 591 (2006)
8. L. Maiani, R. Petronzio, E. Zavattini, Phys. Lett. B **175**, 359 (1986)
9. G. Raffelt, L. Stodolski, Phys. Rev. D **37**, 1237 (1988)
10. E. Massó, J. Redondo, JCAP **0509**, 015 (2005) [hep-ph/0504202]
11. E. Massó, Phys. Rev. Lett. **97**, 151802 (2006) [hep-ph/0606163]
12. G. Jarlskog, L. Jönsson, S. Prünster, H.D. Schulz, H.J. Willutzki, G.G. Winter, Phys. Rev. D **8**, 3813 (1973)
13. H. Persson, I. Lindgren, L.N. Labzowsky, G. Plunien, T. Beier, G. Soff, Phys. Rev. A **54**, 2805 (1996)
14. S. Mallampalli, J. Sapirstein, Phys. Rev. Lett. **80**, 5297 (1998)
15. E. Zavattini et al., Phys. Rev. Lett. **96**, 110406 (2006) [hep-ex/0507107]
16. J.T. Mendonça, J. Dias de Deus, P. Castelo Ferreira, Phys. Rev. Lett. **97**, 100403 (2006)
17. E. Zavattini et al., Phys. Rev. Lett. **97**, 269901(E) (2006) [hep-ph/0606099]
18. S.L. Adler, J. Phys. A **40**, F143 (2007) [hep-ph/0611267]
19. S. Biswas, K. Melnikov, Phys. Rev. D **75**, 053003 (2007) [hep-ph/0611345]
20. M. Roncadelli, in XII International Workshop on Neutrino Telescopes 2007, arXiv:0706.4244
21. H. Gies, W. Dittrich, Phys. Lett. B **431**, 420 (1998) [hep-ph/9804303]
22. W. Dittrich, H. Gies, Phys. Rev. D **58**, 025004 (1998) [hep-ph/9804375]
23. M. Ahlers, H. Gies, J. Jaeckel, A. Ringwald, Phys. Rev. D **75**, 035011 (2007) [hep-ph/0612098]
24. J.T. Mendonça, Eur. Phys. Lett. **79**, 21001 (2007)
25. A.I. Nikishov, Zh. Eksp. Teor. Fiz. **41**, 549 (1961)
26. A.I. Nikishov, Sov. Phys. JETP **14**, 393 (1962)
27. P. Goldreich, P. Morrison, Zh. Eksp. Teor. Fiz. **45**, 344 (1963)
28. P. Goldreich, P. Morrison, Sov. Phys. JETP **18**, 239 (1964)
29. R.J. Gould, G. Schréder, Phys. Rev. Lett. **16**, 252 (1966)
30. R.J. Gould, G. Schréder, Phys. Rev. **155**, 1404 (1967)
31. R.J. Gould, G. Schréder, Phys. Rev. **155**, 1408 (1967)

32. F.W. Stecker, Nature **220**, 675 (1968)
33. F.W. Stecker, Nature **224**, 870 (1969)
34. F.W. Stecker, Phys. Rev. **180**, 1264 (1969)
35. F.W. Stecker, Nature **226**, 135 (1969)
36. F.W. Stecker, J. Silk, Nature **221**, 1229 (1969)
37. S.L. Adler, C. Shubert, Phys. Rev. Lett. **77**, 1695 (1996)
38. A.K. Harding, M.G. Baring, P.L. Gonthier, Astrophys. J. **476**, 246 (1997)
39. M.G. Baring, A.K. Harding, Astrophys. J. **482**, 372 (1997)
40. S.L. Adler, Astrophys. J. **547**, 929 (2001)
41. S. Lee, Phys. Rev. D **58**, 043 004 (1998) [astro-ph/9604098]
42. J.A. Grifols, E. Massó, R. Toldrà, Phys. Rev. Lett. **77**, 2372 (1996) [astro-ph/9606028]
43. J.-L. Han, K. Ferriere, R.N. Manchester, Astrophys. J. **610**, 820 (2004) [astro-ph/0404221]
44. CANGAROO-II Collaboration, K. Tsuchiya et al., Astrophys. J. **606**, L115 (2004) [astro-ph/0403592]
45. VERITAS Collaboration, K. Kosack et al., Astrophys. J. **608**, L97 (2004) [astro-ph/0403422]
46. MAGIC Collaboration, J. Albert et al., Astrophys. J. **638**, L101 (2006) [astro-ph/0512469]
47. C. Csáki, N. Kaloper, M. Peloso, J. Terning, JCAP **0305**, 005 (2003) [hep-ph/0302030]
48. A. De Angelis, O. Mansutti, M. Roncadelli, arXiv: 0707.2695
49. A. De Angelis, O. Mansutti, M. Roncadelli, arXiv: 0707.4312
50. A. Mirizzi, G.G. Raffelt, P.D. Serpico, Phys. Rev. Lett. **95** [astro-ph/0506078]
51. A. Mirizzi, G.G. Raffelt, P.D. Serpico, astro-ph/0506078
52. A. Mirizzi, G.G. Raffelt, P.D. Serpico, arXiv:0704.3044
53. HESS Collaboration, F. Aharonian et al., Astron. Astrophys. **425**, L13 (2004) [astro-ph/0408145]
54. HESS Collaboration, Phys. Rev. Lett. **97**, 221 102 (2006), Erratum-ibid. **97**, 249 901 (2006) [astro-ph/0610509]
55. D. Bersier et al., Astrophys. J. **583**, L63 (2003) [astro-ph/0206465]
56. W. Coburn, S.E. Boggs, Nature **423**, 415 (2003) [astro-ph/0305377]
57. J. Granot, Astrophys. J. **596**, L17 (2003) [astro-ph/0306322]
58. C. Itziken, J.-B. Zuber (McGraw-Hill Book Company, New York, 1980) (see section 4-3)
59. W.-M. Yao et al., Review of Particle Physics, J. Phys. G **33** (2006)
60. A.G. Grozin, Y.F. Pinelis, Mod. Phys. Lett. A **3**, 725 (1988)
61. A.G. Grozin, Y.F. Pinelis, Z. Phys. C **33**, 419 (1987)
62. A.G. Grozin, Phys. Lett. B **166**, 429 (1986)
63. H. Fusaoka, Y. Koide, hep-ph/9706211
64. H. Fusaoka, Y. Koide, Phys. Rev. D **57**, 3986 (1998) [hep-ph/9712201]
65. S. Weinberg, Phys. Rev. Lett. **18**, 188 (1967)
66. S. Weinberg, Phys. Rev. **166**, 1568 (1968)
67. S. Coleman, J. Wess, B. Zumino, Phys. Rev. **177**, 2239 (1969)
68. C. Callan, S. Coleman, J. Wess, B. Zumino, Phys. Rev. **177**, 2247 (1969)
69. R. Dashen, Phys. Rev. **183**, 1245 (1969)
70. R. Dashen, M. Weinstein, Phys. Rev. **183**, 1261 (1969)
71. L.-F. Li, H. Pagels, Phys. Rev. Lett. **26**, 1024 (1971)
72. H. Pagels, Phys. Rep. C **16**, 219 (1975)
73. S. Weinberg, Physica A **96**, 327 (1979)
74. J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984)
75. H. Leutwyler, Ann. Phys. **235**, 165 (1994) [hep-ph/9311274]
76. S.L. Adler, C.G. Callan, D.J. Gross, R. Jackiw, Phys. Rev. D **6**, 2982 (1972)
77. I.A. Shushpanov, A.V. Smilga, Phys. Lett. B **402**, 351 (1997) [hep-ph/9703201]
78. N.O. Agasian, I.A. Shushpanov, Phys. Lett. B **472**, 143 (2000) [hep-ph/9911254]
79. D. Kabat, K.-M. Lee, E. Weinberg, Phys. Rev. D **66**, 014 004 (2002) [hep-ph/0204120]
80. Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)
81. Y. Nambu, G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961)
82. S.P. Klevansky, R.H. Lemmer, Phys. Rev. D **39**, 3478 (1989)
83. P.J.A. Bicudo, J.E.F.T. Ribeiro, Phys. Rev. D **42**, 1611 (1990)
84. K.G. Klimenko, Teor. Mat. Fiz. **89**, 211 (1991)
85. K.G. Klimenko, Theor. Math. Phys. **89**, 1161 (1992)
86. K.G. Klimenko, Z. Phys. C **54**, 323 (1992)
87. D. Ebert, K.G. Klimenko, M.A. Vdovichenko, A.S. Vshivtsev, Phys. Rev. D **61**, 025 005 (2000) [hep-ph/9905253]
88. R. Gonzalez Felipe, G.M. Marques, J.E. Ribeiro, Nucl. Phys. A **778**, 30 (2006) [hep-th/0307290]
89. V.L. Ginzburg, L.D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950)
90. L.N. Cooper, Phys. Rev. **104**, 1189 (1956)
91. J. Bardeen, L.N. Cooper, J.R. Schrieffer, Phys. Rev. **106**, 162 (1957)
92. J. Bardeen, L.N. Cooper, J.R. Schrieffer, Phys. Rev. **108**, 1175 (1957)
93. E.V. Beveren, F. Kleefeld, G. Rupp, M.D. Scadron, Mod. Phys. Lett. A **17**, 1673 (2002) [hep-ph/0204139]
94. V. Radhakrishnan, D.J. Cooke, Astrophys. Lett. **3**, 225 (1969)
95. D. Bhattacharyaa, E.P.J. van den Heuvel, Phys. Rep. **203**, 1 (1991)
96. A.K. Harding, Science **251**, 1033 (1991)
97. R.C. Duncan, C. Thompson, Astrophys. J. **392**, 9 (1992)
98. C. Kouveliotou et al., Nature **393**, 235 (1998)
99. A. Spitkovsky, astro-ph/0310731
100. J.M. Lattimer, M. Prakash, Science **304**, 536 (2004)
101. D.J. Price, S. Rosswog, Science **312**, 719 (2006)
102. J.-P. De Villiers, J.F. Hawley, J.H. Krolik, astro-ph/0307260
103. S. Hirose, J.H. Krolik, J.-P. De Villiers, J.F. Hawley, Astrophys. J. **606**, 1083 (2004) [astro-ph/0311500]
104. J.-P. De Villiers, J.F. Hawley, J.H. Krolik, S. Hirose, Astrophys. J. **620**, 878 (2005) [astro-ph/0407092]
105. J.H. Krolik, J.F. Hawley, S. Hirose, Astrophys. J. **622**, 1008 (2005) [astro-ph/0409231]